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VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examination 2022

(Under CBCS Pattern)

Semester - VI

Subject : MATHEMATICS

Paper : C 13 - T

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

[METRIC SPACES AND COMPLEX ANALYSIS]

Group - A

1. Answer any ***five*** questions : 2×5=10

(a) Let $A=\{(x,y): x,y\in\mathbb{R},\ x\notin\mathbb{Q}\ or\ y\notin\mathbb{Q}\}$. Examine whether A is compact or not in $\mathbb{R}^2$ with the usual metric.

(b) Consider $\mathbb{R}^2$ with the usual metric. Examine whether the set

$A=\left\{\left(x,\sin\frac{1}{x}\right)\in\mathbb{R}^2 : 0<x<1\right\}$ is connected or not.

(c) Define finite intersection property. Does the collection $A=\{(n-1,n+1):n\in Z\}$ of open in intervals satisfy finite intersection property? Justify.

(d) Show that every Cauchy sequence in a metric space is bounded.

(e) Find the radius of convergence of the power series $\sum_{n=1}^{\infty}(4+3i)^n z^n$.

(f) Let $T(z)=\dfrac{az+b}{cz+d}$ be a bilinear transformation. Show that ∞ is a fixed point of T if and only if $c=0$.

(g) Let $f'(z)=2x+ixy^2$ where $z=x+iy$. Show that $f'(z)$ does not exist at any point of z-plane.

(h) Show that $f(z)=e^{-|z|^4}+z+5$ is not differentiable at any non-zero point.

2. Answer any ***four*** questions : 5×4=20

(a) (i) Prove that a metrix space (X, d) having the property that every continuous map $f:X\to X$ has a fixed point, is connected. 2

(ii) Let (X, d) be a complete metric space and $T:X\to X$ be a contraction on X. Then for $x\in X$, show that the sequence $\{T^n(x)\}$ is a convergent sequence. 3

(b) Let (X, d_1) and (Y, d_2) be two metric spaces and $f:(X,d_1)\to(Y,d_2)$ be uniformly continuous. Show that if $\{x_n\}$ is a Cauchy sequence in (X, d_1) then so is $\{f(x_n)\}$ in (Y, d_2). Is it true if f is only continuous? Justify. 3+2

(c) Show that continuous image of a compact metric space is compact. 5

(d) Let $f(z)=u+iv$ be analytic in a domain D. Prove that f is constant in D if and only if one of the following holds :

(i) $f'(z)$ vanishes in D.

(ii) *Re f (z) = u = constant.*

(iii) *Imf (z) = v = constant.*

(iv) $|f(z)|$ = *constant (non zero)*

Check the analyticity of $f(z)=\overline{z}$. 5

(e) Find the domain of convergence of the following series

$$\sum_{n=1}^{\infty}\frac{1.3.5.....(2n-1)}{n!}\left(\frac{1-z}{z}\right)^n .$$ 5

(f) Evaluate :

(i) $\int_C \frac{\sin(\pi z^2)+\cos(\pi z^2)}{(z-1)(z-2)}dz$ where C is the circle $|z| = 3$ described in the positive sense.

(ii) $\int_C \frac{zdz}{(9-z^2)(z+i)}$, where C is the circle $|z|=2$ described in the positive sense.

2+3

3. Answer any ***three*** questions : 10×3=30

(a) (i) Let (X, d) be a metric space and A be a compact subset of X. Show that A is totally bounded.

(ii) A subset A of a metric space (X, d) is totally bounded if and only if every sequence in A has a Cauchy sequence.

(iii) If $\mathcal{F}$ be a family of compact sets with finite intersection property in a metric space (X, d), then show that $\cap\mathcal{F}\neq\phi$. 2+5+3

(b) (i) Show that the map $f:[0,1]\to[0,1]$, defined by $f(x)=x-\frac{x^2}{2}, x\in(0,1)$ is a weak contraction but not a contraction map.

(ii) Let (X, d) be a complete metric space and $f: X\to X$ be a contraction map with Lipschitz constant $t(0<t<1)$. If $x_0\in X$ is the unique fixed point of f, show that $d(x, x_0) \leq \frac{1}{1-t} d(x, f(x))$, for all $x \in X$.

(iii) Show that a contraction of a bounded plane set may have the same diameter as the set itself. 3+5+2

(c) (i) Let $f(z) = u(x, y) + iv(x, y), z = x + iy$ and $z_0 = x_0 + iy_0$. Let the function f be defined in a domain D except possible at the point z_0 in D. Then prove that $\lim_{z \to 0} f(z) = u_0$ if and only if $\lim_{x \to x_0} u(x, y) = u_0$ and $\lim_{y \to y_0} v(x, y) = v_0$.

(ii) If $f(z) = u(x, y) + iv(x, y)$ is an analytic function of $z = x + iy$ and $u(x, y) - v(x, y) = \frac{e^y - \cos x + \sin x}{\cosh y - \cos x}$ find *f(z)* subject to the condition $f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}$. 5+5

(d) (i) Suppose *f(z)* is analytic in a domain Ω and $C = \{z : |z - a| = R\}$ contained in Ω. Then prove that $\left|f^n(a)\right| \leq \frac{n! M_R}{R^n}, n = 0, 1, 2, ...$

where $M_R = \max_{z \in C} |f(z)|$.

(ii) Show that every bounded entire function is constant.

(iii) Let $p(z) = a_0 + a_1 z + a_2 z^2 + ... + a_n z^n, a_n \neq 0$. Show that there exists a point z_0 in C such that $p(z_0) = 0$. 3+3+4

(e) (i) Show that when $0 < |z| < 4$, $\frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}$.

(ii) Find the Laurent series that represents the function $f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$ in the domain $0 < |z| < \infty$. 5+5

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(Under CBCS Pattern)

Semester - VI

Subject : MATHEMATICS

Paper : C 14 - T

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
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[RING THEORY AND LINEAR ALGEBRA-II]

1. Answer any ***five*** questions : 2×5=10

(a) Show that $1+\sqrt{-5}$ is irreducible element in $Z\left(\sqrt{-5}\right)$.

(b) Let *a, b* be two nonzero elements of an Euclidean domain *R*. If *b* is not a unit in *R*, then prove that $d(a)<d(ab)$.

(c) Give an example (with justification) of a division ring which is not a field.

(d) Determine all the associates of [8] in the ring $\mathbb{Z}_{10}$.

(e) Give an example of a linear operator *T* on a finite dimensional vector space *V* over a field *F* such that *T* is not diagonalizable.

(f) Find the *dual basis* of the basis {(1, 1, 2), (1, 0, 1), (2, 1, 0)} of the vector space $\mathbb{R}^3$.

(g) If a real symmetric matrix is positive definite then show that all its eigen values are positive.

(h) If $T \in A(V)$ and S is regular in $A(V)$, prove that T and STS^{-1} have same minimal polynomial, where $A(V)$ is the annihilator of V.

2. Answer any ***four*** questions : 5×4=20

(a) (i) Prove that 1 and –1 are the only units of the ring $\mathbb{Z}\sqrt{-5}$.

(ii) Show that the integral domain $\mathbb{Z}\sqrt{-5}$ is a factorization domain. 3+2=5

(b) Find *gcd* of 11 + 7*i* and 18 – *i* in *Z* + *iZ*.

(c) Let $T : V \to V$ be a linear mapping, where V is a Euclidean space. Show that T is orthogonal if and only if T maps an orthogonal basis to an orthonormal basis.

(d) Let V be a finite dimensional vector space over the field F and T be a diagonalizable linear operator on V. Let $\{c_1, c_2, \ldots\ldots, c_k\}$ be the set of all distinct eigen values of T. Then prove that the characteristic polynomial of T is of the form $(x-c_1)^{d_1}(x-c_2)^{d_2}\ldots.(x-c_k)^{d_k}$ for some positive integers $d_1, d_2, \ldots.., d_k$.

(e) (i) Let T be a linear operator on a finite dimensional vector space V over F. Define minimal polynomial of T.

(ii) If V is finite dimensional over F, then prove that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial of T is not 0, where $A(V)$ is the annihilator of V. 1+4

(f) Find the eigen values and bases for the eigen space of the matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{pmatrix}$.

Is A diagonalizable?

3. Answer any ***three*** questions : 10×3=30

(a) (i) Let R be a PID. Prove that p is irreducible in R if and only if the ideal generated by p is a non-zero maximal ideal. Hence show that $\mathbb{Q}[x]/<x^2-2>$ is a field.

(ii) Prove that for any linear operator T on a finite-dimensional inner product space V, there exists a unique linear operator T^* on V such that $<T\alpha, \beta> = <\alpha, T^*\beta>$ for all $\alpha, \beta \in V$. (4+2)+4=10

(b) (i) Let N be a 2×2 complex matrix such that $N^2 = 0$. Then prove that either $N = 0$ or N is similar to the matrix $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ over $\mathbb{C}$.

(ii) Use Gram-Schmidt process to obtain an orthonormal basis from the following basis $\mathcal{B} = \{ (1, 2, -2), (2, 0, 1), (1, 1, 0)\}$ of $\mathbb{R}^3$ with the standard inner product. 4+6=10

(c) (i) Show that an element x in a Euclidean domain is a unit if and only if $d(x) = d(1)$. Hence find all units in the ring $Z + iZ$ of Gaussian integers.

(ii) Define unique factorization domain (UFD). Show that $R = \{a + b\sqrt{-5} \mid a, b \in Z\}$ is not UFD. 6+4

(d) (i) Consider the polynomial $f(x) = 5x^4 + 4x^3 - 6x^2 - 14x + 2$ in $\mathbb{Z}[x]$. Using Eisenstein's criterion show that $f(x)$ is irreducible in $\mathbb{Z}$.

(ii) Let $A = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & 1 & 0 \end{pmatrix}$. Find its minimal polynomial over $\mathbb{R}$ and hence check whether A is similar to a diagonal matrix or not.

(iii) Consider the inner product space $\mathbb{C}^2$ over $\mathbb{C}$ with the standard inner product. Let T be a linear operator on $\mathbb{C}^2$ such that the matrix representation of T with respect to the standard ordered basis is $A = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$. Show that T is a normal operator. 3+(3+1)+3=10

(e) (i) Let a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$T(x, y, z) = (2x + y - 2z, 2x + 3y - 4z, x + y - z)$. Find all eigen values of T and find a basis of each eigen space.

(ii) The matrix of a linear mapping $f : \mathbb{R}^3 \to \mathbb{R}^3$ relative to the standard basis is $\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \end{vmatrix}$. Find f and its matrix with respect to the basis $\{(0,1,-1),(1,-1,1),(-1,1,0)\}$. 5+5

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Question Paper

B.Sc. Honours Examinations 2020

(Under CBCS Pattern)

Semester - VI

Subject: MATHEMATICS

Paper: DSE - 3

(Mechanics – Theory)

(Number Theory – Theory)

(Industrial mathematics – Theory)

Full Marks: 60 (Theory)

Time: 3 Hours (Theory)

Candiates are required to give their answer in their own words as far as practicable.

Questions are of equal value.

Answer any **one question** from the following:

Mechanics (Theory)

1. (a) What is astatic equilibrium? Define the astatic center and prove that it is independent of angle of rotation of forces.

   (b) A perfectly rough plane is inclined at an angle α to the horizon; show that the least eccentricity of the ellipse which can rest on the plane is $\sqrt{\dfrac{2\sin\alpha}{1+\sin\alpha}}$

2. (a) State and prove the principle of virtual work.

(b) A force P acts along the axis of X and another force nP acts along the generator of the cylinder $x^2 + y^2 = a^2$; show that the central axis lies on the cylinder

$$n^2(nx - z)^2 + (1 + n^2)^2 y^2 = a^2 n^4.$$

(a) A planet of mass M and periodic time T when at its greatest distance from the sun comes into collision with a meteor of mass m, moving in the same orbit in the opposite direction with velocity v. If $\frac{m}{M}$ be small, then show that the major axis of the planet's path is reduced by $\frac{4m}{M} \cdot \frac{vT}{\pi} \sqrt{\frac{1-e}{1+e}}$

(b) A particle is projected vertically upwards with a velocity u on a medium whose resistance varies as the square of the velocity. Investigate the motion.

4. (a) Find the radial and cross-radial components velocity and acceleration of a moving particle.

(b) A heavy particle is projected with a velocity V from the end of a horizontal diameter of a sphere of radius a along the inner surface, the direction of projection making an angle β with the equator. If the particle never leaves the surface, then prove that

$$3\sin^2\beta < 2 + \left(\frac{V^2}{3ag}\right)^2$$

5. (a) Prove that the rate of change of angular momentum of the body about the axis of rotation is equal to the sum of the moments about the same axis of all forces acting on the body.

(b) Show that the moment of inertia of a right solid cone of mass M whose height is h and radius of whose base a is $\frac{3Ma^2}{20} \cdot \frac{6h^2 + a^2}{h^2 + a^2}$ about a slant side and $\frac{3M}{80} \cdot (h^2 + a^2)$ about a straight line through the centre of gravity of the cone perpendicular to its axis.

6. (a) Show that the total kinetic energy of a rigid body moving in two dimensions

$$\frac{1}{2}Mv^2 + \frac{1}{2}Mk^2\dot{\theta}^2$$

and angular momentum is $Mvp + Mk^2\dot{\theta}$; symbols have their usual meaning.

(b) An elliptic area, of eccentricity e is rotating with angular velocity ω about one latus rectum. Suddenly, this latus rectum is loosed and the other is fixed. Show that the new angular velocity is $\frac{1-4e^2}{1+4e^2}\omega$.

7. (a) A system of coplanar forces acting on a rigid body. Find the conditions under which the given system of forces can be compounded into a single resultant force only, and obtain the equation of the line of action of the single resultant force in this case.

(b) A uniform bowl has its inner and outer surfaces concentric hemisphere with radii a and b respectively; prove that the distance of its c.g. from the centre is $\frac{3(a+b)(a^2+b^2)}{8(a^2+ab+b^2)}$.

8. (a) A perfectly rough body rests in equilibrium on a fixed body. The parts of the bodies in contact are spherical and the surface of the lower body is concave upwards, the upper body is in equilibrium at the lowest. Stating the necessary condition of equilibrium, establish the relation for stable equilibrium.

(b) Six equal heavy rods each of weight w, freely hinged at their ends, form a regular hexagon ABCDEF which when hung up by the points A is kept from altering the shape by two light rods BF and CE. Prove that the thrust of the light rods are $\frac{5\sqrt{3}}{2}W$ and $\frac{\sqrt{3}W}{2}$.

9. (a) Forces X, Y, Z are acting along three straight lines $y=b, z=-c$; $z=c, x=-a$; $x=a, y=-b$ respectively; find the condition that they have a single resultant. Hence find the equation of the line of action of the system.

(b) A solid hemisphere of weight W rests in limiting equilibrium with its curved surface on a rough inclined plane and the face is horizontal by a weight P attached at a point in the rim. Find the coefficient of friction between the hemisphere and the plane.

10. (a) Two masses M, m, are connected by a string which passes through a hole in a smooth horizontal table, m hanging vertically. Show that M describes a curve whose differential equation is $\left(1+\frac{m}{M}\right)\frac{d^2u}{d\theta^2}+u=\frac{mg}{M}\frac{1}{h^2u^2}$. If M describes nearly a circular orbit with centre as the hole, then show that the apsidal angle is $\pi\sqrt{\frac{M+m}{3M}}$.

(b) A particle of mass m is projected vertically upward, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is $\frac{V^2}{g}\left[\lambda - \log(1+\lambda)\right]$ after a time $\frac{V}{g}\log(1+\lambda)$ where V is the terminal velocity of the particle and λV is the initial velocity of the particle.

11. (a) What is mean by principal axis of a given material system of a point? Find whether given straight line is at any point of its length a principal axis of the material system and in case the line is the principal axis find the other two principal axis.

(b) A plank of mass M is initially at rest along a straight line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man, of mass M' starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$ where a is the length of the plank and g is the acceleration due to gravity.

12. (a) A rough uniform rod, of length $2a$, is placed on a rough table at right angles to its edge. If the centre of gravity of the rod be initially at a distance b beyond the edge, then show that the rod will begin to slide when it has turned through an angle $\tan\theta = \frac{\mu a^2}{a^2+9b^2}$, where μ is the coefficient of friction.

(b) A solid homogeneous cone of height h and vertical angle oscillates about a horizontal axis through its vertex. Show that the length of the simple equivalent pendulum is .

[The symbols have their usual meaning]

Number Theory (Theory)

1. (a) If x, y, z is a primitive Pythagorean triple, prove that $x+y$ and $x-y$ are congruent modulo 8 to either 1 or 7.

   (b) Show that the radius of the inscribed circle of a Pythagorean triangle is always an integer.

   (c) Show that the Fermat equation $x^n + y^n = z^n$ has no solution in the positive integers for all $n\left(=2^k\right)$ where $k > 2$ is any positive integer.

   (d) If a and b are relatively prime positive odd integers, each greater than 1, then show that .

   $(a/b).(b/a) = (-1)^{\frac{a-1}{2}\frac{b-1}{2}}$.

   (e) Find all positive integers such that n^2+1 is divisible by $n+1$.

2. (a) What is plaintext and ciphertext?

   (b) The ciphertext X IBCS EBWUBWKIRYU has been enciphered with the cipher $C_1 \equiv 4P_1 + 11P_2 \pmod{26}$, $C_2 \equiv 3P_1 + 8P_2 \pmod{26}$. Derive the plaintext.

   (c) If p is an odd prime and $(a, p) = 1$, then the congruence $x^2 \equiv a \left(\bmod p^n\right)$ has a solution if and only if $(a/p) = 1$.

   (d) Let p be an odd prime. Show that the Diophantine equation $x^2 + py + a = 0$, $\gcd(a, p) = 1$ has an integral solution if and only if $(-a/p) = 1$. Determine whether $x^2 + 7y - 2 = 0$ has a solution in the integers.

   (e) Prove that if for some integers a, b, c we have $9 \mid a^3 + b^3 + c^3$, then at least one of the numbers a, b, c is divisible by 3.

3. (a) Let p be an odd prime and let $\gcd(a, p) = 1$. If n denotes the number of integers in the set $S = \left\{a, 2a, 3a, \ldots, \left(\frac{p-1}{2}\right)a\right\}$ whose remainders upon division by p exceed $\frac{p}{2}$, then $(a/p) = (-1)^n$.

(b) Let p be an odd prime and $(a, p) = \gcd(b, p) = 1$. Prove that either all three of the quadratic congruences $x^2 \equiv a \pmod p$, $x^2 \equiv b \pmod p$, $x^2 \equiv ab \pmod p$ are solvable or exactly one of them admits a solution.

(c) If r is a primitive root of the odd prime p, show that

$$ind_r(p-a) \equiv ind_r a + \frac{(p-1)}{2} \pmod{p-1}.$$

(d) Find the remainder when $2^{23}.7^{13}$ is divided by 17.

4. (a) Prove that if p is an odd prime number and $k \geq 1$, then there exists a primitive root for p^k.

(b) For an odd prime p, verify that the sum

$$1^n + 2^n + 3^n + \ldots + (p-1)^n = \begin{cases} 0 \pmod p \ \ if \ (p-1) \mid n \\ -1 \pmod p \ \ if \ (p-1) \mid n \end{cases}$$

(c) Determine the exact number of primitive roots of an integer n (if exists).

(d) If m and n are relatively prime positive integers, prove that

$$m^{\phi(n)} + n^{\phi(m)} \equiv 1 \pmod{mn}$$

(e) State and prove Fermat's little theorem.

5. (a) Define Goldbach conjecture. Show that the Goldbach conjecture implies that for each even integer $2n$ there exist integers n_1 and n_2 with $\phi(n_1) + \phi(n_2) = 2n$.

(b) If $n = p_1^{k_1} p_2^{k_2} \ldots p_r^{k_r}$ where $p_1, p_2, \ldots, p_r$ are distinct primes, then show that $\tau(n)\phi(n) \geq n$.

(c) If n be an integer and $1 \leq r < n$ then show that $n(n-1)(n-2)\ldots(n-r+1)$ is an integer.

(d) State and prove Mobius inversion formula.

(e) For any positive integer n, show that $\dfrac{\sigma(n!)}{n!} \geq 1 + \dfrac{1}{2} + \dfrac{1}{3} + \ldots + \dfrac{1}{n}$.

6. (a) One egg remains when the eggs are removed from the basket 2, 3, 4, 5, or 6 at a time; but, no eggs remain if they are removed 7 at a time. Find the smallest number of eggs that could have been in the basket.

(b) For $n > 3$, show that the integers $n, n+2, n+4$ cannot all be prime.

(c) Every integer of the form $n^4 + 4$, with $n > 1$, is composite.

(d) Divide 100 into two summands such that one is divisible by 7 and the other by 11.

(e) Find the unit digit of 7^{140}.

(f) Find the number of solutions in ordered pairs of positive integers (x, y) of the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}$ where n is a positive integer.

7. (a) Find the integral solution of $68x - 157y = 1$.

(b) If n is a positive integer and p a prime number then prove that the exponent of the highest power of p that divides $n!$ is $\sum_{k=1}^{\infty}\left[\frac{n}{p^k}\right]$ where the series is finite, because $\left[\frac{n}{p^k}\right] = 0$ for $p^k > n$.

8. (a) If the integer a has order k modulo n and $h > 0$, then show that a^h has order $\frac{k}{\gcd(h, k)}$ modulo n.

(b) Find the order of 119 modulo 18.

(c) Find two primitive roots of 10.

9. (a) If $m = 2^h, h > 2$ and $\gcd(a, m) = 1$ then show that $a^{\frac{\phi(m)}{2}} \equiv 1 (\bmod m)$.

(b) If p is a prime number and $d \mid p-1$, then show that the congruence $x^d - 1 \equiv 0 \ (\bmod p)$ has exactly d solutions.

10. (a) Solve $5x^2 + 6x + 1 \equiv 0 \ (\bmod 23)$.

(b) Let p be an odd prime and $\gcd(a,\ p)=1$. Show that a is a quadratic residue of p if and only if $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$.

11. (a) Define Legendre symbol.

(b) Examine whether $x^2 \equiv -46 \pmod{17}$ is solvable.

(c) Use Gauss's lemma to compute Legendre symbol of (i) $\left(\frac{7}{13}\right)$ and (ii) $\left(\frac{11}{23}\right)$.

12. (a) Find three different Pythagorean triplets, not necessarily primitives of the form 16, y, z.

(b) Solve the linear congruence relation $140x \equiv 133 \pmod{301}$.

(c) Find the highest power of 5 dividing 1000!

[The symbols have their usual meaning]

Industrial Mathematics (Theory)

1. Explain why Beer's law is a plausible model for X-ray attenuation.

2. Determine and explain the Radon transform of a phantom composed of elliptical regions, each having a constant attenuation coefficient.

3. (a) Define Back projection of R. Verify that, for given values of a, b, and θ, the line $l_{(a\cos\theta+b\sin\theta),\theta}$ passes through the point (a, b).

   (b) Find the Fourier transform of $f(x) = e^{-\lambda|x|}\cos(\omega_0 x)$.

   (c) For all $a \in R$ show that $\int_{-\infty}^{\infty} f(x)\delta(x-a)\,dx = f(a)$.

4. Discuss a short note on ART and how it is useful in CT scan.

5. Write a short note on inversion problems and how inversion formula is useful in precalculus using an example.

6. Write a short note on medical imaging and what are the applications of mathematics in medical imaging.

7. (a) Explain different important areas in industrial mathematics.

   (b) Show that, for all choices of t and ? and all suitable functions f,

   $$Rf(t,\theta) = Rf(-t, \theta+\pi)$$

8. (a) Can CT scans diagnose COVID-19? Give your justification.

   (b) The line $1_{1/2,\,\pi/6}$ has the standard parameterization $x = \frac{\sqrt{3}}{4} - \frac{s}{2}$ and $y = \frac{1}{4} + \frac{\sqrt{3}s}{2}$ for $-\infty < s < \infty$. Find the values of s at which this line intersects the unit circle.

9. Discuss different applications of various properties of Fourier transform and inverse Fourier transform in image reconstruction.

10. Explain different geological anomalies in earth's interior from measurements at its surface and Tomography. How can medical imaging be distinguished from Inverse Problems?

11. Discuss important properties of 'black projection' with different examples.

12. Discuss the algorithms of CT scan machine.

[The symbols have their usual meaning]

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Question Paper

B.Sc. Honours Examination 2022

(Under CBCS Pattern)

Semester - VI

Subject : MATHEMATICS

Paper : DSE 3 - T

Full Marks : 60

Time : 3 Hours

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The figures in the margin indicate full marks.

[MECHANICS]

1. Answer any ***five*** questions : 2×5=10

(a) What are stable and unstable equilibrium ? State energy test of stability.

(b) State the principle of virtual work for a system of forces acting on a rigid body.

(c) State Kepler's laws of planetary motion.

(d) If the anglular velocity about the origin be a constant w, then find the cross-radial component of rate of change of acceleration of the particle.

(e) If X, Y be the algebraic sums of the resolved parts of all the forces acting in a plane along the rectangular axes θ_x and θ_y; and G be the algebraic sum of the moments of the forces about O; find the locus of the points at which the algebraic sum of the moments of the forces is constant and is equal to G^1.

(f) Define 'apse' of a central orbit. Show that, at an apse, a particle is moving at right angles to the radius vector of the point.

(g) An artificial satelite revolves about the earth at a height H above the surface. Find the orbital speed, so that a man in the satelite will be in a state of weightlessness.

(h) State D' Alembert's principle. Write down the general equations of motion of a rigid body.

2. Answer any ***four*** questions : 5×4=20

(a) Find the co-ordinates of C.G. of a lamina in the shape of a quadrant of the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1$, density at (x, y) is $\rho = kxy$, where k is constant.

(b) A square lamina rests with its plane perpendicular to a smooth wall, one corner being attached to a point in the wall by a fine string of length equal to the side of the square. Find the position of equilibrium and show that it is stable.

(c) At the vertex C of a triangle ABC which is right angled at C, show that the principal axes are a perpendicular to the plane and two others inclined to the sides at an angle $\frac{1}{2}\tan^{-1}\frac{ab}{a^2-b^2}$.

(d) An ellipse of axes a, b and a circle of radius b are cut from the same sheet of a uniform metal and are suspended and fixed together with their centres coincident. The figure is free to move in its own vertical plane about one end of its major axis. Show that the length of the equivalent simple pendulum is $\frac{5a^2-ab+2b^2}{4a}$.

(e) A particle is projected at right angles to the line joining it to a centre of force, attracting according to the law of inverse square of the distance, with a velocity $\frac{\sqrt{3}}{2}V$, where V denotes the velocity from infinity. Find the eccentricity of the orbit described and show that the periodic time is $2\pi T$; T being the time taken to describe the major-axis of the orbit with velocity V.

(f) Find the accelerations of a particle, moving in 3-dimensional space, in terms of polar co-ordinates.

3. Answer any ***three*** questions : 10×3=30

(a) (i) The middle points of opposite sides of a quadrilateral formed by four freely jointed weightless bars are connected by two light rods of length 'a' and 'b' in a state of tension. If T_1 and T_2 be the tensions of those rods, prove that $\frac{T_1}{a}+\frac{T_2}{b}=0$.

(ii) A surface is formed by revolution of rectangular hyperbola about a vertical asymptote; show that a particle will rest on it everywhere beyond its intersection with a certain circular cylinder. 5+5

(b) (i) If X, Y, Z, L, M, N are six components of a system of forces, deduce the invariants of the system.

(ii) Equal forces act along the axes and along the straight line $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$. Find the equation of the central axis of the system.

5+5

(c) (i) A particle moves with a central acceleration $\{\mu \div (\text{distance})^2\}$. It is projected with a velocity V at a distance R. Show that its path is a rectangular hyperbola, if the angle of projection is $\sin^{-1}\left\{\frac{\mu}{VR\sqrt{V^2-\frac{2\mu}{R}}}\right\}$.

(ii) One end of an elastic string, of unstretched length 'a', is tied to a point on a smooth table and a particle is attached to the other end and can move freely on the table. If the path be nearly a circle of radius b, then show that apsidal angle is approximately $\pi\sqrt{\frac{b-a}{4b-3a}}$. 5+5

(d) (i) A thin rod of length $2a$ revolves with uniform angular velocity ω about a vertical axis through a small joint at one extremity of the rod, so that it describes a cone of semi-vertical angle α. Show that $\omega^2=\frac{3g}{4a\cos\alpha}$.

(ii) An elliptic lamina is such that when it swings about one latus rectum as a horizontal axis, the other latus rectum passes through the centre of oscillation. Prove that the eccentricity of the ellipse is $\frac{1}{2}$. 5+5

(e) (i) A particle of Unit mass is projected with velocity u at an inclination α about the horizon in a medium whose resistance is k times the velocity. Show that the direction of the path described will again make an angle α with the horizon after a time $\frac{1}{k}\log\left\{1+\frac{2ku}{g}\sin\alpha\right\}$.

(ii) Find the apsidal angle in a nearly circular orbit under the central force $ar^m + br^n$; a, b are constants. 6+4

OR

[NUMBER THEORY]

1. Answer any ***five*** questions : 2×5=10

(a) If a is odd integer then show that $32 \mid (a^2 + 7)(a^2 + 3)$.

(b) Show that 41 divides $2^{20} - 1$.

(c) If $n > 1$ is an integer not of the form $6k + 3$, prove that $n^2 + 2^n$ is composite.

(d) Let $\tau(n)$ denote the number of positive divisors of n and $\sigma(n)$ denote the sum of these divisors. Find $\tau(160)$ and $\sigma(160)$.

(e) Define Legendre symbol. Find the value of the Legendre symbols 9/23.

(f) If p and q are odd primes satisfying $p = q + 4a$ for some a, establish that $\left(\frac{a}{p}\right) = \left(\frac{a}{q}\right)$.

(g) Let $a_n = 6^n + 8^n$. Determine the remainder on dividing a_{83} by 49.

(h) For what valus of n does $n!$ terminates in 37 zeros?

2. Answer any ***four*** questions : 5×4=20

(a) Prove that the integer $53^{103} + 103^{53}$ is divisible by 39, and that $111^{333} + 333^{111}$ is divisible by 7.

(b) Determine all solutions in the positive integers of the Diophantine equations : $123x + 360y = 99$.

(c) Let $n = p_1^{k_1}, p_2^{k_2} .....p_r^{kr}$ be the prime factorization of the integer $n > 1$. If f is a multiplicative function that is not identically zero, prove that

$$\sum_{d/n} \mu(d) f(d) = (1 - f(p_1))(1 - f(p_2)) ......(1 - f(p_r))$$

(d) If $n \geq 1$ and p is a prime, prove that the exponent of the highest power of p that divides $(2n)!(n!)^2$ is $\sum_{k=1}^{\infty}\left(\left[\frac{2n}{p^k}\right] - 2\left[\frac{n}{p^k}\right]\right)$.

(e) Let p be an odd prime and gcd (a, p) = 1. Then a is a quadratic residue of p if and only if $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$.

(f) Encrypt the plaintext message GODL MEDAL using the RSA algorithm with key (n, k) = (2419, 3).

3. Answer any ***three*** questions : 10×3=30

(a) (i) Show that all the solutions of the Pythagorean equation $x^2 + y^2 = z^2$ satisfying the conditions $\gcd(x, y, z) = 1, 2 \mid x,\ x > 0, y > 0, z > 0$ are given by the formulas $x = 2st,\ y = s^2 - t^2,\ z = s^2 + t^2$ for integers s > t > 0 such that gcd (s, t) = 1 and $s \neq t \pmod{2}$. 5

(ii) Find all positive integers n such that $3^{n-1} + 5^{n-1} \mid 3^n + 5^n$. 3

(iii) If a primitive root exists for a positive integer n, then determine the exact number of primitive root for n. 2

(b) (i) Let p be an odd prime and let gcd (a, p) = 1. If n denotes the number of integers in the set $S = \left\{a, 2a, 3a, \ldots.., \left(\frac{p-1}{2}\right)a\right\}$.

whose remainders upon division by exceed $\frac{p}{2}$, then show that $\left(\frac{a}{p}\right) = (-1)^n$.

(ii) If m and n are relatively prime positive integers, prove that $m^{\varphi(n)} + n^{\varphi(m)} \equiv 1$ (mod mn). 5

(c) Given a positive integer n, let $\tau(n)$ denote the number of positive divisors of n and $\sigma(n)$ denote the sum of these divisors. If $n = p_1^{k_1} p_2^{k2} \ldots. p_r^{k_r}$ is the prime factorization of $n > 1$, then prove that

(i) $\tau(n) = (k_1 + 1)(k_2 + 1) \ldots. (k_r + 1)$, and

(ii) $\sigma(n) = \frac{p_1^{k_1+1} - 1}{p_1 - 1} \frac{p_2^{k_2+1} - 1}{p_2 - 1} \ldots. \frac{p_r^{k_r+1} - 1}{p_r - 1}$

(d) Let $n = a_m(1000)^m + a_{m-1}(1000)^{m-1} + ... + a_1(1000) + a_0$ where a_k are integers and $0 \le a_k \le 999, k = 0,1,......., m$ be the representation of a positive integer n.

Let $T = a_0 - a_1 + a_2 - ...... + (-1)^m a_m$. Then

(i) n is divisible by 7 if and only if T is divisible by 7.

(ii) n is divisible by 11 if and only if T is divisible by 11.

(iii) n is divisible by 13 if and only if T is divisible by 13. 3+3+4

(e) (i) Let p be an odd prime and $gcd(a, p) = 1$. Then prove that a is a quadratic residue of p if and only if $a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$. 5

(ii) If $p = 2^k + 1$ is prime, verify that every quadratic non residue of p is a primitive root of p. 3

(iii) Find the value of (–72/131). 2

OR

[INDUSTRIAL MATHEMATICS]

1. Answer any ***five*** questions : $2 \times 5 = 10$

   (a) Write the full form of MRI.

   (b) What is the full form of CT?

   (c) Who invented CT scan?

   (d) Mention two limitations of CT.

   (e) What is computed tomography

   (f) What is a scatter radiation?

   (g) Which CT imagers is often referred to as the heart scan?

   (h) Is the radiation from an X-Ray dangerous?

2. Answer any ***four*** questions : $5 \times 4 = 20$

   (a) Discuss various types of medical imaging.

   (b) What is inverse problem in image processing?

   (c) Discuss the concept behind the city scan.

   (d) How image is formed in CT scan?

   (e) Discuss various types of X-Ray machines.

   (f) Discuss the properties of back projection.

3. Answer any ***three*** questions : $10 \times 3 = 30$

   (a) How do you solve inverse problems?

   (b) Define random transform with an example. How is it used to obtain the projections of object? 4+6

   (c) How does iterative reconstruction work in CT?

   (d) Discuss the reconstruction methods for CT imaging.

   (e) Discuss geological anomalies in Earth's interior from measurements at its surface.

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বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examination 2022

(Under CBCS Pattern)

Semester - VI

Subject : MATHEMATICS

Paper : DSE 4 - T

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

[MATHEMATICAL MODELLING]

1. Answer any ***five*** questions : 2×5=10

   (a) Find the Laplace transform of the function $F(t-a)$, where $F(t-a)=\begin{cases}0, t \geq a \\ 0, t < a\end{cases}$.

   (b) If $L^{-1}\{f(p)\}=f(t)$, then prove that $L^{-1}\left\{\int_s^{\infty} f(u)\,du\right\}=\frac{F(t)}{t}$.

   (c) Verify the initial value theorem for the function $(2t+3)^2$.

(d) Show that although (2, 3, 2) is a feasible solution to the system of equations

$$x_1 + x_2 + 2x_3 = 9$$
$$3x_1 + 2x_2 + 5x_3 = 22$$
$$x_1, x_2, x_3 \geq 0$$

it is not a basic solution.

(e) Use middle-square method to generate 2 random numbers considering the seed x_0 = 1009.

(f) At work station, 5 jobs arrive every minute. The mean time spent on each job in the work station is $\frac{1}{8}$ minute. What is the mean steady state number of jobs in the system?

(g) Cars arrive at a service station according to Poisson's distribution with a mean rate of 5 per hour. The service time per car is exponential with a mean of 10 minutes. What is the average waiting time at steady state in the queue?

(h) Show that $x = 0$ is an ordinary point of the Legendre's differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n\left(n+1\right)y = 0$; where n is a constant.

2. Answer any ***four*** questions : 5×4=20

(a) Solve the following LPP by simplex method :

Maximize $z = -x_1 + 3x_2 - 2x_3$

Subject to

$$\begin{array}{rrrrrrrrr} & 2x_1 & + & 3x_2 & - & x_3 & \leq & 7 \\ - & 2x_1 & & & + & 4x_3 & \leq & 12 \\ & 8x_1 & - & 4x_2 & + & 3x_3 & \leq & 10 \\ & & & x_1, & & x_2, \; x_3 & \geq & 0 \end{array}$$

(b) Using convolution theorem of Laplace transform deduce the formula

$$\int_0^1 t^{a-1}\left(1-t\right)^{b-1} dt = \frac{\Gamma\left(a\right)\Gamma\left(b\right)}{\Gamma\left(a+b\right)}, a > 0, b > 0$$

(c) Food X contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and costs 12 paise per gram. Food Y contains 8 units of vitamin A per gram, 12 units of vitamin B per gram and costs 20 paise per gram. The daily minimum requirements of vitamin A and B are 100 units and 120 units respectively. Formulate the problem as a LPP model and find the minimum costs of the product mixture.

(d) What do you mean cycling in linear congruence. Use the linear congruence method to generate 20 random numbers using $a = 5$, $b = 3$ and $c = 16$.

(e) Using Laplace transform solve the following initial-value problem

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-2t}, y(0) = 1, y'(0) = 0$$

(f) Solve $\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}$ $x > 0, t > 0$ using Laplace transform where $y(0, t) = 1$, $y(x, 0) = 0$.

3. Answer any ***three*** questions : 10×3=30

(a) (i) Discuss the use of Monte Carlo simulation to model a deterministic behavior; the area under a curve.

(ii) Write an algorithm to calculate an approximation to π using Monte Carlo simulation, considering the random number selected inside the quarter circle $Q : x^2 + y^2 = 1,\ x \geq 0,\ y \geq 0$

where Q lies inside the square.

$S : 0 \leq x \leq 1,\ 0 \leq y \leq 1$

Use the equation $\frac{\pi}{4} = \frac{are\,Q}{are\,S}$. 5+5

(b) Consider the following LPP

Maximize $z = 3x_1 + 5x_2$

Subject to
$$\begin{array}{cccl} x_1 & & & \leq 4 \\ 3x_1 & + & 2x_2 & \leq 18 \\ & x_1, & x_2 & \geq 0 \end{array}$$

If a new variable x_5 is introduced with cost $c_5 = 3$ and corresponding vector $a_5 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$; Discuss the effect of adding the new variable and obtain the revised solution if any.

10

(c) (i) Find $L^{-1}\left\{\frac{1}{\sqrt{s}(s-a)}\right\}$.

(ii) Find the solution of the Bessel differential equation of order λ at the neighborhood of $x = 0$. Discuss the case when $\lambda = 0$. 3+7

(d) (i) If $L^{-1}\{F(s)\} = f(t)$ and $L^{-1}\{G(s)\} = g(t)$ then prove that

$$L^{-1}\{F(s)G(s)\} = \int_0^t f(\tau)g(t-\tau)\,d\tau$$

(ii) Using the result of (i), find $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+4s+5)}\right\}$. 5+5

(e) (i) Prove the Final value theorem $\underset{t\to\infty}{Lt}\, f(t) = \underset{s\to 0}{Lt}\, sF(s)$.

(ii) Solve the following LPP by graphical method

Maximize $z = 10x + 35y$

Subject to 4+6

$$\begin{aligned} 8x + 6y &\le 48 \\ 4x + y &\le 20 \\ y &\ge 5 \\ x,\ y &\ge 0 \end{aligned}$$

OR

[DIFFERENTIAL GEOMETRY]

1. Answer any ***five*** questions : 2×5=10

   (a) Define Fundamental Plane.

   (b) Define surface and curvilinear coordinates.

   (c) Find the equation of the tangent plane of $\sigma(r,\theta)=\left(r\cosh\theta, r\sinh\theta, r^2\right)$ at (1, 1, 1).

   (d) Find the asymptotic line on the surface $z = y\sin x$.

   (e) Define developable.

   (f) What do you mean by helix?

   (g) Define geodesics.

   (h) Define surface curve.

2. Answer any ***four*** questions : 5×4=20

   (a) Show that the asymptotic lines of the hyperboloid $\vec{r} = a\cos\theta\sec\psi\hat{i} + b\sin\theta\sec\psi\,\hat{j} + c\tan\psi\hat{k}$ are given by $\theta\pm\psi$ = constant.

   (b) Prove that the geodesics on a right circular cylinder are helices.

   (c) State and prove Euler's theorem on normal curvature. 1+4

   (d) Find the first fundamental magnitudes for the curve $\vec{r} = \left(u\cos v, u\sin v, cv\right)$.

   (e) Derive tangential and polar developable associated with a space curve.

   (f) Show that asymptotic lines on the Paraboloid $2z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ are $\frac{x}{a} \pm \frac{y}{b} =$ constant.

3. Answer any ***three*** questions : 10×3=30

(a) (i) Show that the curves $u + v =$ constant, are geodesics on a surface with metric $\left(1+u^2\right)du^2 - 2uvdudv + \left(1+v^2\right)dv^2$.

(ii) Derive the partial differential equation of surface theory. 5+5

(b) (i) Prove that the second fundamental form at any point of the surface has the value which equals twice the length of the perpendicular from continuous point to a point on the tangent plane.

(ii) Discusses the geometric significance of the second fundamental form. 6+4

(c) (i) Define torsion and geodesic curvature. Derive analytical representation of geodesic curvature.

(ii) Find the curvature and torsion of the curve

$x = a\left(u - \sin u\right), y = a\left(1 - \cos u\right), z = bu$ 1+1+4+2+2

(d) For the helicoids $z = c\tan^{-1}\frac{y}{x}$, prove that $P_1 = -P_2 = \frac{u^2+c^2}{c}$, where $u^2 = x^2 + y^2$ and the lines of curvature are given by $d\theta = \pm\frac{du}{\sqrt{u^2+c^2}}$ and $z = c\theta$. 5+5

(e) (i) State and prove Gauss-Bonnet theorem.

(ii) Find the geodesics on the ellipsoid of the revolution $\frac{x^2+z^2}{a^2} + \frac{y^2}{b^2} = 1$.

1+4+5

OR

[BIO MATHEMATICS]

1. Answer any ***five*** questions : 2×5=10

(a) The fish growth model by Von Bertalanffy is given by $\frac{dF(t)}{dt} = \alpha F^{\frac{3}{2}}(t) - \beta F(t)$, where $F(t)$ denotes the weight of the fish and α, β are positive constants. Discuss the stability at the equilibrium point.

(b) Define Lyapunov stable and uniformly stable.

(c) Consider a stock of fish that is being harvested at a constant rate $\frac{dN}{dt} = f(N) - h$, where $f(N) = rN\left(1 - \frac{N}{K}\right)$. What is the maximum sustainable yield for sufficiently small harvest levels h?

(d) Discuss the Allee effect given by $\frac{dN}{dt} = N\left[k_0 - l(N-\mu)^2\right], \mu < \sqrt{\frac{k_0}{l}}$ where k_0 and μ are positive constants.

(e) Define the term immigration and emigration.

(f) What are the physical significance of the dominant eigen value of the Leslie matrix?

(g) What is the saddle-node bifurcation of the system $\frac{dy}{dx} = f(x, \mu)$?

(h) Write short note on Routh-Hurwitz criterion.

2. Answer any ***four*** questions : 5×4=20

(a) Determine the nature of critical point (0, 0) of the system $\frac{dx}{dt} = 2x - 7y$, $\frac{dy}{dt} = 3x - 8y$. Also check the stability of the system at critical point.

(b) Write a short note on Nicholson-Bailey host parasite model.

(c) For the system $\frac{dx}{dt} = x - y - x\left(x^2 + y^2\right)$, $\frac{dy}{dt} = x + y - y\left(x^2 + y^2\right)$, check the existence of a limit cycle.

(d) Discuss about Holling's type functional response.

(e) Using the concept of S-I-R model, formulate a Covid-19 pandemic model, with out considering vaccination. State all the parameters clearly.

(f) Consider the flow of fluid due to pressure gradient in a tube of radius *a* and length *l*. Find the bounds for the velocity distribution.

3. Answer any ***three*** questions : 10×3=30

(a) (i) Formulate differential equation and find steady state solution of SIR (Susceptible-Immigration-Removal) epidemic model.

(ii) Discuss the stability of the steady state solution of that. 5+5

(b) (i) Find out the steady state solution and discuss the stability of the prey-predator model $\frac{dN_1}{dt} = N_1\left(\alpha - \beta N_2\right)$; $\alpha > 0,\ \beta > 0$;

$$\frac{dN_2}{dt} = -N_2\left(\gamma - \delta N_1\right); \gamma > 0, \delta > 0$$

where N_1 and N_2 respresent the density of prey and density of predator respectively.

(ii) Give the geometrical interpretation of the above prey-predator model. 6+4

(c) The delayed Lotka-Volterra competition system is given by

$$\frac{dx(t)}{dt} = x(t)\left[2 - \alpha x(t) - \beta y(t-r)\right]; \alpha > 0,\ \beta > 0$$

$$\frac{dy(t)}{dt} = y(t)\left[2 - yx(t-r) - \delta y(t)\right]; \gamma > 0,\ \delta > 0$$

(i) Obtain the steady-state solutions (if exist).

(ii) Investigate the stability of the non-zero steady-states for $\alpha = \delta = 2$ and $\beta = \gamma = 1$. 4+6

(d) (i) Deduce Fisher's equation for spreading of genes.

(ii) What are the additional restrictions on Fisher's problem for traveling wave solution? 5+5

(e) Let N(t) be the number of tiger population at any time t. The quotient of birth rate and death rate by the population size N are respectively by,

$$\frac{\text{Birthrate}}{\text{N}} = \frac{3}{2} + \frac{1}{1000}\text{N} \text{ and } \frac{\text{Deathrate}}{\text{N}} = \frac{1}{2} + \frac{1}{3000}\text{N}.$$

Formulate a model (using differential equation) that describes the growth and regulation of this tiger population. Solve for N(t), assuming N(0)=100 and describe the long term behavior of this tiger population as $t \to \infty$. 3+4+3